# High-Precision Simulations of the Evolving Universe to Weigh Cosmic Neutrinos

In July 2021, a six-dimensional Vlasov simulation was executed using nearly all the nodes of supercomputer Fugaku. Researcher Kohji Yoshikawa became the first person in the world to develop software that could successfully perform such a computation. His results showed, with much greater precision than previously possible, how the distribution of neutrinos throughout the universe has changed from the Big Bang up to the present day. How did he arrive at this achievement? And what is the scientific significance of these results?

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## Neutrinos Influence the Formation of the Large-Scale Structure of the Universe

The countless galaxies throughout the universe are known to not be distributed uniformly but rather have a structure similar to sticking bubbles. Our current understanding is that galaxies are clustered along the surfaces of many enormous structures conceived as "bubbles", with relatively few galaxies residing inside these bubbles. In the very early days of our universe, matter was distributed mostly uniformly, albeit with some slight fluctuation variations in density. We think the gravity of the slightly denser regions attracted nearby matter, thereby causing them to grow and attract ever more matter.

This difference in density gradually increased, creating the *large-scale structure of the universe* as it now exists.

"Dark matter" is believed to have played a major role in the evolution of the universe. Dark matter is an unidentified and invisible component of the universe, only known for exerting gravity on surrounding matter. Although we can't see dark matter, we believe it makes up *most of the mass in the universe* and that its distribution throughout the universe corresponds to the observed distribution of galaxies. To better understand how the large-scale structure of our universe came to be, researchers run "time evolution" simulations and study how the distribution of dark matter *evolves over time*. In such simulations, researchers have adopted a well-established method called "N-body" simulation, in which the distribution of matter in the universe is represented by a very large number of particles mutually interacting via gravity.

Around the year 2000, it was discovered that even tiny neutrinos have some amount of mass. Scientists then began to include neutrinos in their simulations of the evolving universe. "Neutrinos are extremely light, but they're quite abundant in the universe. They zip around at tremendous speeds in the universe, which interferes with the dark matter's ability to attract mass gravitationally. So, scientists started running N-body simulations that include neutrinos, but they weren't accurate enough," explains Yoshikawa.

## Taking on an Equation Once Thought Unsolvable

Rather than N-body simulations, Yoshikawa opted for Vlasov simulations, which solve the "Vlasov equation". Here's how they differ.

Neutrinos hurtle through the universe at various speeds and directions. In computer simulations, the positions and velocities of these neutrinos are represented by coordinates of a virtual "phase space" which consists of a 3D position space and a 3D velocity space, meaning six dimensions in total. This is the case in both of N-body and Vlasov simulations (Fig. 1a). The difference is that in N-body simulations, the virtual "particles" are placed in a neutrino-dense area and then their changes in position and velocity over time are computed (Fig. 1b). In Vlasov simulations, on the other hand, the phase space is divided into fine mesh grids and the *time evolution* of the neutrino distribution within those mesh grids is computed (Fig. 1c). Whereas real-world neutrino distributions are quite smooth

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**Fig. 1**  **Vlasov Simulations and N-body Simulations of Neutrino Distribution in the Universe**

Simulations of the evolving universe compute the motion of matter in a virtual "phase space" in the form of 3D coordinates for both position and velocity. Real-world neutrinos are smoothly distributed in the phase space (a). In N-body simulations (b), however, these distributions are represented by discrete particles. In Vlasov simulations (c), the neutrino distribution represented in the form of a distribution function mapped onto the fine mesh grids is very smooth. So, the Vlasov simulations produce less noisy, more accurate results than the N-body simulations.



(a) Distribution of neutrinos in phase space

(b) N-body simulations　　(c) Vlasov simulations
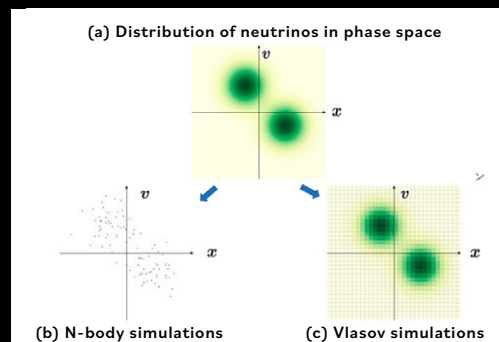
(Fig. 1a), the scattered particle images produced by the N-body simulations contain a significant amount of random noise. The Vlasov simulations can properly compute even low-density areas, allowing for more accurate results.

The actual results of Yoshikawa's simulations (Figure 2) show that Vlasov simulations can reproduce the smooth distribution of neutrinos. But, solving six-dimensional Vlasov equations requires a supercomputer with enormous memory capacity and exceptional performance, not to mention a numerical algorithm that is both very efficient and very accurate. Yoshikawa's results overturned commonly-held long-standing assumptions that such computations were impossible.

Yoshikawa began developing the numerical code for these simulations in 2007, when he started working at the University of Tsukuba. After several years of trial and error, in 2013 he successfully demonstrated that six-dimensional Vlasov equations could be solved on the university's "T2K-Tsukuba" supercomputer. This was a world's first achievement. Next, at the suggestion of Naoki Yoshida at the University of Tokyo, Yoshikawa decided to use his simulation code to tackle the real-world problem of neutrino distribution in the universe over time.

However, developing a "numerical scheme" to solve these 6D Vlasov equations for the expanding universe with sufficient numerical accuracy was not easy.
 Yoshikawa reflects "I thought about quitting it many times." Takashi Minoshima at the Japan Agency for Marine-Earth Science and Technology provided him with an important clue to overcoming the difficulty. The two promptly began working together. By 2017, their numerical schemes had been improved and become much more accurate. Yoshikawa performed numerical simulations with the improved numerical scheme on a variety of supercomputers, including the K computer. Then in the fall of

**Fig. 2      Comparing Vlasov Simulations with N-body Simulations**

Comparison of neutrino distributions obtained with a Vlasov simulation (a) and an N-body simulation with a similar computational cost (b).
In the Vlasov simulation, the neutrino distribution is smooth, whereas the counterpart in the N-body simulation appears somewhat pixelated, making the subtleties of the structures harder to be discerned.



(a) Vlasov simulation          (b) N-body simulation
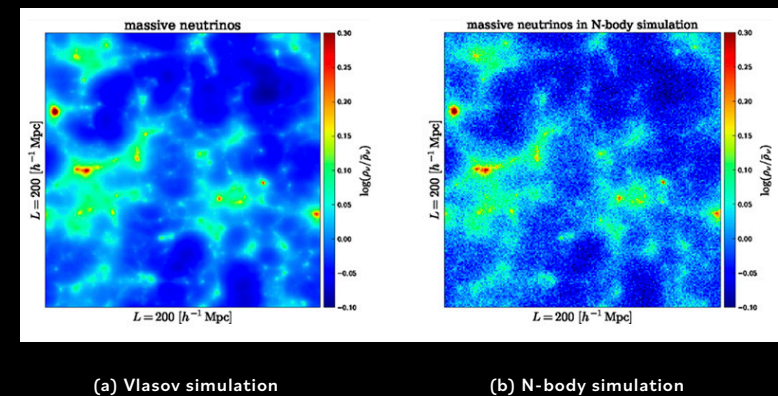
2019, he was given permission to run his simulations on an early access system of Fugaku.

## All-node Calculations Get Maximum Performance out of Fugaku

Fugaku's huge memory space, a fast interconnect and high parallelization efficiency make it ideal for Vlasov simulations, in which calculations on a large number of mesh grids are performed in parallel. However, since Fugaku has a SIMD (Single Instruction Multiple Data) instruction set different from previous supercomputers, the code has to be refactored to match the Fugaku's instruction set for the maximum performance on Fugaku. Yoshikawa devised an idea for speeding up the calculations, and postdoctoral researcher Satoshi Tanaka (currently a research fellow at the Yukawa Institute for Theoretical Physics, Kyoto University) implemented it in the program.

In July 2021, a Vlasov simulation of cosmic neutrinos adopting about 400 trillion mesh grids was successfully performed using 147,456 nodes, or about 93% of the total 158,976 nodes on Fugaku (Fig. 3a).

This was, of course, the largest Vlasov simulation ever conducted in the world. It was also 10 times faster than a comparable N-body simulation previously performed on a Chinese supercomputer.

The distributions of neutrinos and dark matter are computed with Vlasov and N-body simulations, respectively (Fig 3b). "We can constrain the possible range of yet-to-be-known neutrino mass by analyzing the simulated data and comparing them with observational data.", explains Yoshikawa about the scientific significance of these simulations. Weighing the neutrino mass directly leads to the understanding of the dynamical effect of neutrinos on the large-scale structure formation in the universe. Furthermore, the Vlasov simulations are expected to be successfully applied to the numerical simulations of magnetic plasma such as "accretion disks" formed around the astrophysical black holes and magnetic storms in the inter-planetary space.

Yoshikawa reflects, "This time, the simulations used practically all the nodes of Fugaku, and also used up nearly all the memory space. There were various issues both in the software and hardware sides.We believe that lots of valuable experiences to overcome such issues will

**Fig. 3** **The Results of Simulations Using 147,456 Nodes on Fugaku**

The time evolution of neutrinos and dark matter was computed using a Vlasov simulation (a) and an N-body simulation (b), respectively. The matter distribution at the very beginning of the universe (about 200 million years after the Big Bang) obtained from actual observations was adopted as the initial condition and evolved into the present-day universe (after the 13.8 billion years after its birth). A timestep of the simulation corresponds to about 2 million years. The size of the simulated volume is 5.6 billion light-years (1200 Mpc/$h$ per) side, which covers nearly all of the observable universe.
[Mpc/$h$ is a unit of length corresponding to about 466 million light years. The $h$ is the Hubble parameter normalized by 100km/s/Mpc and observationally estimated to be around 0.7.]
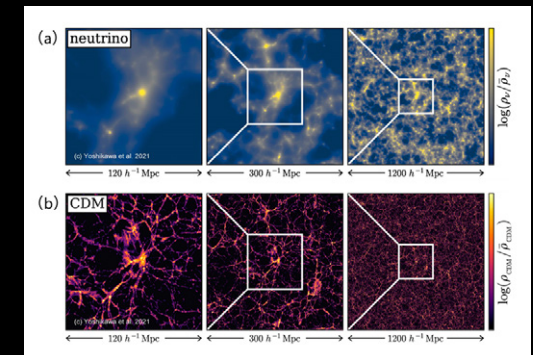
be reflected in the future operation of Fugaku and be useful to bring the best performance it has." This substantial all-node run will be an encouraging impetus to the future utilization of Fugaku.

About the

# Researcher

"I'll become a researcher because I can do what I love for a living." Yoshikawa has always thought so since he was a junior high school student. In 1995, when he was a junior in undergraduate school, he started to study astrophysics attracted by the emerging computer simulations which were not common yet at that time. There, he encountered the Vlasov equation and decided to numerically solve it himself one day. As a mentor, he encourages his students to conduct research with the attitude of doing what no one else is doing. As a native of Suzuka City, which is famous for holding Formula One Grand Prix in Japan, he loves watching motor sports, saying "I love the pursuit of pure speed."
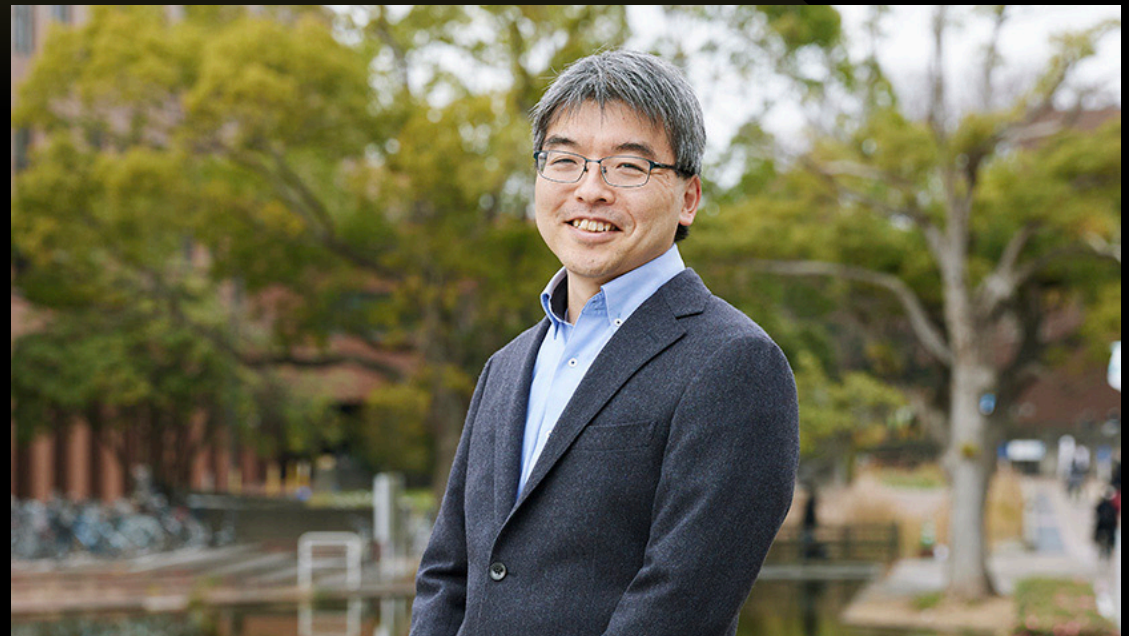


**Associated Research Projects**

• "Toward a Unified View of the Universe: From Large-scale Structures to Planets" (hp200124/hp210164)
Principal Investigator: Junichiro Makino, Kobe University
2021 ACM Gordon Bell Prize finalist for Research Findings

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